Instructions: Try these problems from the textbook as practice (answers below).

Problems

5-12. Use Green's Theorem to evaluate the line integral along the given positively oriented curve.

5. $\displaystyle\int_C ye^x\,dx + 2e^x\,dy$, C is the rectangle with vertices $(0,0)$, $(3,0)$, $(3,4)$ and $(0,4)$
9. $\displaystyle\int_C (y + e^{\sqrt{x}})\,dx + (2x + \cos(y^2))\,dy$, C is the boundary of the region enclosed by the parabolas $y = x^2$ and $x = y^2$
11. $\displaystyle\int_C y^3\,dx - x^3\,dy$, C is the circle $x^2 + y^2 = 4$

13-18. Use Green's Theorem to evaluate $\displaystyle\int_C \mathbf{F}\cdot d\mathbf{r}$. Check the orientation of the curve before applying the theorem!

15. $\mathbf{F}(x,y) = \langle y\cos x - xy\sin x, xy + x\cos x\rangle$, C is the triangle from $(0,0)$ to $(0,4)$ to $(2,0)$ to $(0,0)$
17. $\mathbf{F}(x,y) = \langle y - \cos y, x\sin y\rangle$, C is the circle $(x-3)^2 + (y+4)^2 = 4$ oriented clockwise

23. Find the area under of the cycloid $x = t - \sin t$, $y = 1 - \cos t$.

31. Calculate $\displaystyle\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x,y) = \left\langle \frac{2xy}{(x^2+y^2)^2}, \frac{y^2-x^2}{(x^2+y^2)^2}\right\rangle$ and C is any positively oriented simple colsed curve that encloses the origin.

33. If $\mathbf{F}(x,y) = \left\langle \frac{-y}{(x^2+y^2)^2}, \frac{x}{(x^2+y^2)^2}\right\rangle$ show that $\displaystyle\int_C \mathbf{F}\cdot d\mathbf{r} = 0$ for every simple closed path that does not pass or enclose the origin.

Answers

5. $4(e^3 - 1)$
9. $\dfrac{1}{3}$
11. -24π
15. $-\dfrac{16}{3}$
17. 4π
23. 3π.
31. 0